Deep learning

7.1. Transposed convolutions

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Constructing deep generative architectures requires layers to increase the signal dimension, the contrary of what we have done so far with feed-forward networks.

Some generative processes optimize the input, and as such rely on back-propagation to expend the signal from a low-dimension representation to the high-dimension signal space (e.g. lecture 9.4. "Optimizing inputs")

The same can be done in the forward pass with **transposed convolution layers** whose forward operation corresponds to a convolution layer's backward pass.

Notes

The convolution layers that we have seen until now usually reduce the size of the signal:

- either because the filter size (with no additional padding) reduces the tensor on the outside, or
- because the stride is greater than 1.

Hence they are useful to go from a high dimensional signal (e.g. image, sound sample) to a smaller one (e.g. vector of class scores).
The transposed convolution layers provides a way of increasing the size of the signal, which is necessary for generative tasks.

Consider a 1d convolution with a kernel κ

$$
\begin{aligned}
y_i &= (x \circledast \kappa)_i \\
&= \sum_a x_{i+a-1} \, \kappa_a \\
&= \sum_u x_u \, \kappa_{u-i+1}.
\end{aligned}
$$

We get

$$
\begin{aligned}
\left[\frac{\partial \ell}{\partial x}\right]_u &= \frac{\partial \ell}{\partial x_u} \\
&= \sum_i \frac{\partial \ell}{\partial y_i} \frac{\partial y_i}{\partial x_u} \\
&= \sum_i \frac{\partial \ell}{\partial y_i} \kappa_{u-i+1}.
\end{aligned}
$$

which looks a lot like a standard convolution layer, except that the kernel coefficients are visited in reverse order.

Notes

Since x influences ℓ only through y, we have

$$\frac{\partial \ell}{\partial x_u} = \sum_i \frac{\partial \ell}{\partial y_i} \frac{\partial y_i}{\partial x_u}.$$

We see that

$$\sum_u x_u \, \kappa_{u-i+1}$$

is very similar to

$$\sum_i \frac{\partial \ell}{\partial y_i} \kappa_{u-i+1},$$

except that

- in the first case, the filter κ and the signal x are visited in the same order, as indexed by u, and
- in the second case, the derivative $\frac{\partial \ell}{\partial y}$ and the filter κ are visited in opposite directions, as indexes by i. The filter is "flipped" in this case.

This is actually the standard convolution operator from signal processing. If $*$ denotes this operation, we have

$$(x * \kappa)_i = \sum_a x_a \, \kappa_{i-a+1}.$$

Coming back to the backward pass of the convolution layer, if

$$y = x \circledast \kappa$$

then

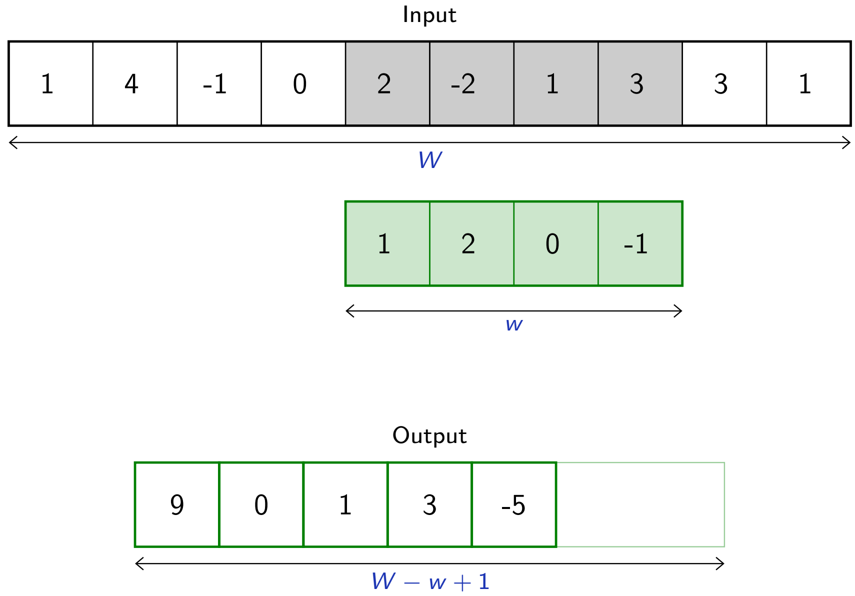
$$\left[\frac{\partial \ell}{\partial x}\right] = \left[\frac{\partial \ell}{\partial y}\right] * \kappa.$$

In the deep-learning field, since it corresponds to transposing the weight matrix of the equivalent fully-connected layer, it is called a **transposed convolution**.

$$
\begin{pmatrix}
\kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 \\
0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 \\
0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 \\
0 & 0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3
\end{pmatrix}^{\top}
=
\begin{pmatrix}
\kappa_1 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_1 & 0 & 0 & 0 \\
\kappa_3 & \kappa_2 & \kappa_1 & 0 & 0 \\
0 & \kappa_3 & \kappa_2 & \kappa_1 & 0 \\
0 & 0 & \kappa_3 & \kappa_2 & \kappa_1 \\
0 & 0 & 0 & \kappa_3 & \kappa_2 \\
0 & 0 & 0 & 0 & \kappa_3
\end{pmatrix}
$$

A convolution can be seen as a series of inner products, a transposed convolution can be seen as a weighted sum of translated kernels.

Convolution layer

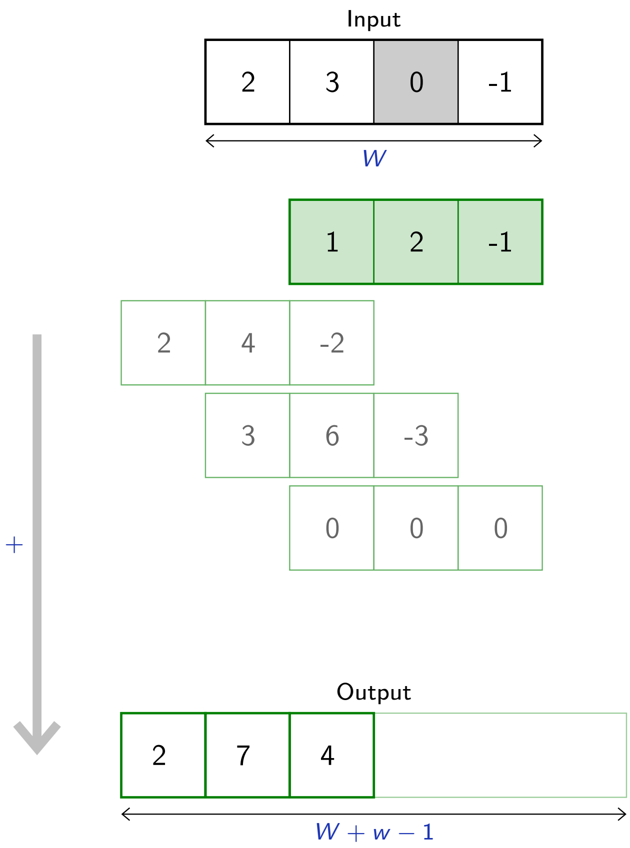


Notes

This convolution can be re-written as the following matrix product

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -1 \\ 0 \\ 2 \\ -2 \\ 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 1 \\ 3 \\ -5 \\ -3 \\ 6 \end{bmatrix}$$

Transposed convolution layer



Notes

This transposed convolution can be formulated as a matrix multiplication as follows:

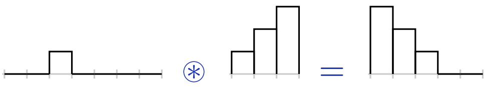
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 4 \\ -4 \\ -2 \\ 1 \end{bmatrix}$$

from which we can interpret as a weighted sum of kernels.
And we also notice that the output dimension is larger then the input one.

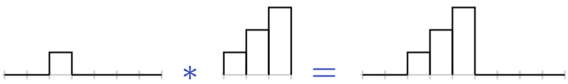
`F.conv_transpose1d` implements the operation we just described. It takes as input a batch of multi-channel samples, and produces a batch of multi-channel samples.

We can compare on a simple 1d example the results of a standard and a transposed convolution:

```
>>> x = torch.tensor([[[0., 0., 1., 0., 0., 0., 0.]]])
>>> k = torch.tensor([[[1., 2., 3.]]])
>>> F.conv1d(x, k)
tensor([[[ 3.,  2.,  1.,  0.,  0.]]])
```



```
>>> F.conv_transpose1d(x, k)
tensor([[[ 0.,  0.,  1.,  2.,  3.,  0.,  0.,  0.,  0.]]])
```



Notes

The transposed convolution increases the signal size and does not flip the filter shape.
So a standard convolution computes at every location of a tensor the responses of linear filters, and a transposed convolution computes at every location a linear combination of kernels.

The class `nn.ConvTranspose1d` embeds that operation into a `nn.Module`.

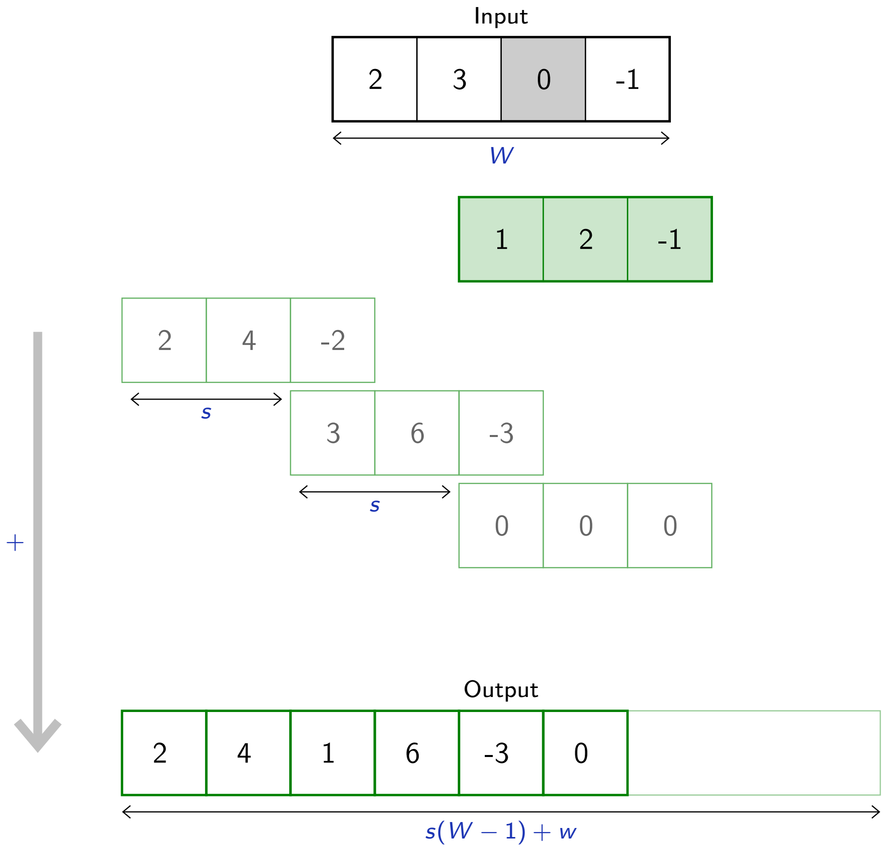
```
>>> x = torch.tensor([[[ 1., 0., 0., 0., -1.]]])
>>> m = nn.ConvTranspose1d(1, 1, kernel_size=3)
>>> with torch.autograd.no_grad():
...   m.bias.zero_()
...   m.weight.copy_(torch.tensor([ 1, 2, 1 ]))
...
Parameter containing:
tensor([0.], requires_grad=True)
Parameter containing:
tensor([[[1., 2., 1.]]], requires_grad=True)
>>> y = m(x)
>>> y
tensor([[[ 1.,  2.,  1.,  0., -1., -2., -1.]]], grad_fn=<SqueezeBackward1>)
```

Transposed convolutions also have a `dilation` parameter that behaves as for convolution and expends the kernel size without increasing the number of parameters by making it sparse.

They also have a `stride` and `padding` parameters, however, due to the relation between convolutions and transposed convolutions:

⚠ While for convolutions `stride` and `padding` are defined in the input map, for transposed convolutions these parameters are defined in the output map, and the latter modulates a cropping operation.

Transposed convolution layer (stride $= 2$)

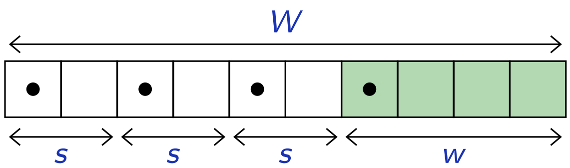


The composition of a convolution and a transposed convolution of same parameters keep the signal size [roughly] unchanged.

A convolution with a stride greater than one may ignore parts of the signal. Its composition with the corresponding transposed convolution generates a map **of the size of the observed area**.

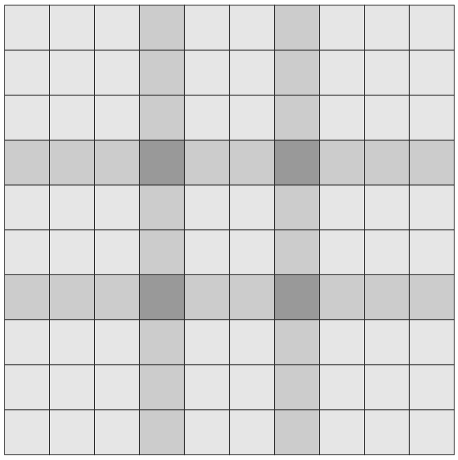
For instance, a 1d convolution of kernel size w and stride s composed with the transposed convolution of same parameters maintains the signal size W, only if

$$\exists q \in \mathbb{N}, \; W = w + s\, q.$$



It has been observed that transposed convolutions may create some grid-structure artifacts, since generated pixels are not all covered similarly.

For instance with a 4×4 kernel and stride 3



Notes

The level of gray of each square is proportional to the number of filters that cover that location. Darker is more visited.

An alternative is to use an analytic up-scaling, implemented in the PyTorch functional `F.interpolate`.

```
>>> x = torch.tensor([[[[ 1., 2. ], [ 3., 4. ]]]])
>>> F.interpolate(x, scale_factor = 3, mode = 'bilinear')
tensor([[[[1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
          [1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
          [1.6667, 1.6667, 2.0000, 2.3333, 2.6667, 2.6667],
          [2.3333, 2.3333, 2.6667, 3.0000, 3.3333, 3.3333],
          [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000],
          [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000]]]])

>>> F.interpolate(x, scale_factor = 3, mode = 'nearest')
tensor([[[[1., 1., 1., 2., 2., 2.],
          [1., 1., 1., 2., 2., 2.],
          [1., 1., 1., 2., 2., 2.],
          [3., 3., 3., 4., 4., 4.],
          [3., 3., 3., 4., 4., 4.],
          [3., 3., 3., 4., 4., 4.]]]])
```

Such module is usually combined with a convolution to learn local corrections to undesirable artifacts of the up-scaling.

In practice, a transposed convolution such as

```
tconv = nn.ConvTranspose2d(nic, noc,
                           kernel_size = 3, stride = 2,
                           padding = 1, output_padding = 1),

y = tconv(x)
```

can be replaced by

```
conv = nn.Conv2d(nic, noc, kernel_size = 3, padding = 1)

u = F.interpolate(x, scale_factor = 2, mode = 'bilinear')
y = conv(u)
```