Deep learning

3.3. Linear separability and feature design

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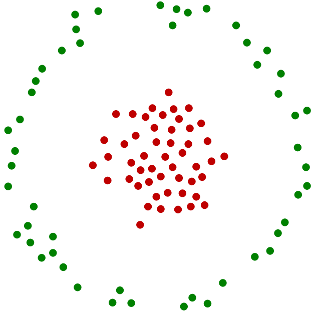

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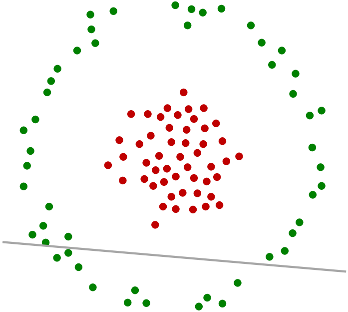

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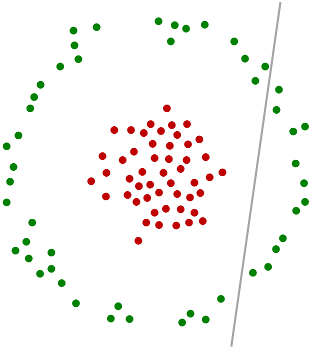

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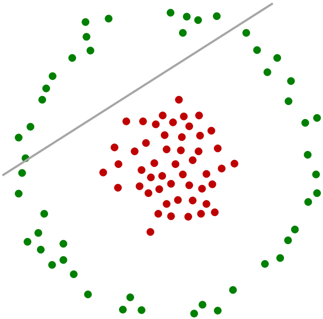

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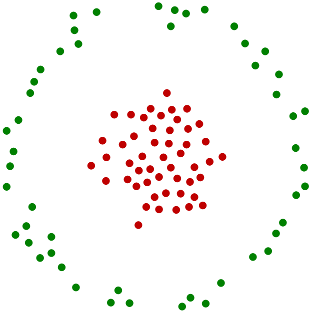
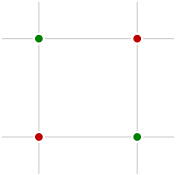
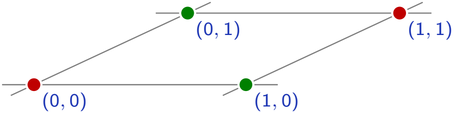




“xor”

The xor example can be solved by pre-processing the data to make the two populations linearly separable.



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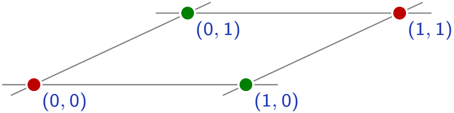
$$\Phi : (x_u, x_v) \mapsto (x_u, x_v, x_u x_v).$$



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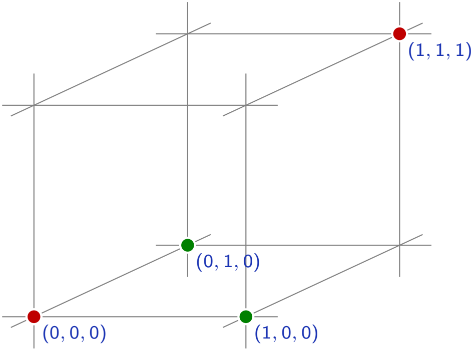

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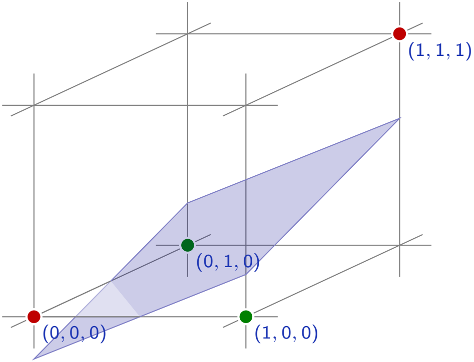

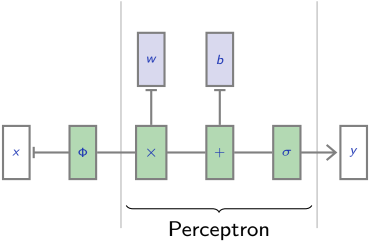
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Perceptron

This is similar to the polynomial regression. If we have

$$\Phi : x \mapsto (1, x, x^2, \dots, x^D)$$

and

$$\alpha = (\alpha_0, \dots, \alpha_D)$$

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By increasing D, we can approximate any continuous real function on a compact space (Stone-Weierstrass theorem).

It means that we can make the capacity as high as we want.

We can apply the same to a more realistic binary classification problem: MNIST's "8" vs. the other classes with a perceptron.

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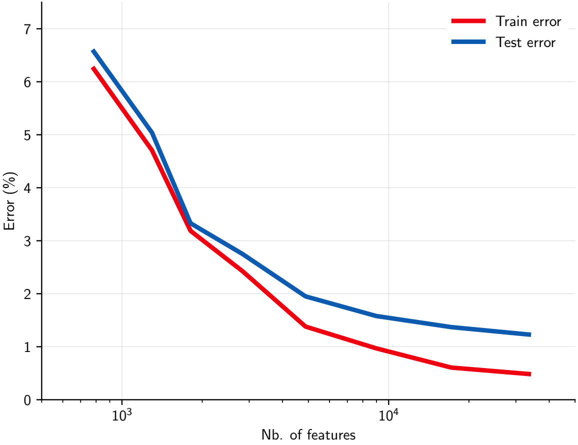

Remember the bias-variance tradeoff we saw in 2.3. “Bias-variance dilemma”

$$\mathbb{E}((Y-y)^2) = \underbrace{(\mathbb{E}(Y)-y)^2}_{\text{Bias}} + \underbrace{\mathbb{V}(Y)}_{\text{Variance}}.$$

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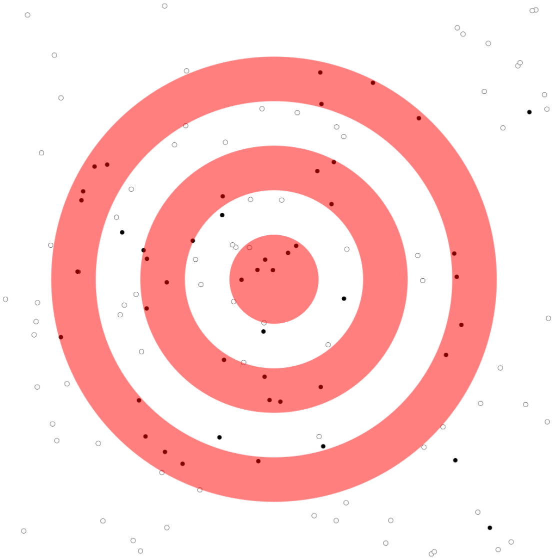
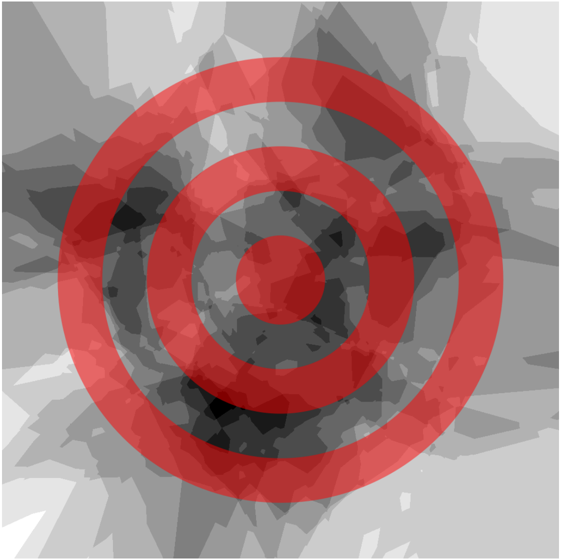
In particular, good features should be invariant to perturbations of the signal known to keep the value to predict unchanged.

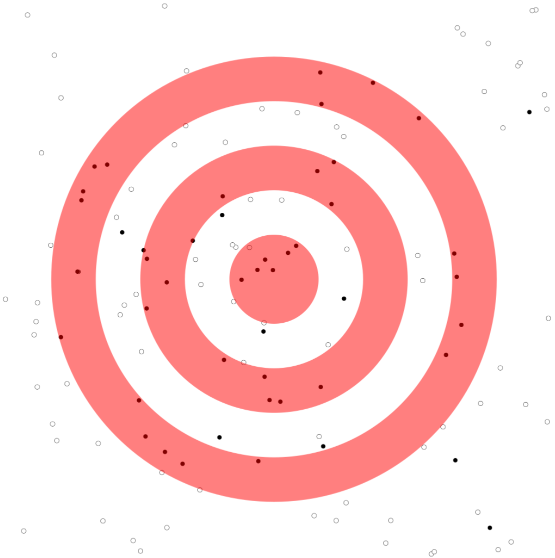


Training points



Votes (K=11)

Prediction (K=11)



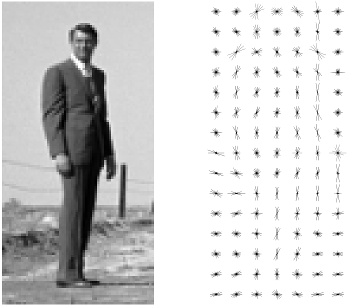
Training points

Votes, radial feature (K=11)

Prediction, radial feature (K=11)

A classical example is the "Histogram of Oriented Gradient" descriptors (HOG), initially designed for person detection.

Roughly: divide the image in 8×8 blocks, compute in each the distribution of edge orientations over 9 bins.



Dalal and Triggs (2005) combined them with a SVM, and Dollár et al. (2009) extended them with other modalities into the "channel features".

Many methods (perceptron, SVM, k-means, PCA, etc.) only require to compute $\kappa(x, x') = \Phi(x) \cdot \Phi(x')$ for any (x, x').

So one needs to specify κ alone, and may keep Φ undefined.

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This is the **kernel trick**, which we will not talk about in this course.

Training a model composed of manually engineered features and a parametric model such as logistic regression is now referred to as **"shallow learning"**.

The signal goes through a single processing trained from data.

The end

References

N. Dalal and B. Triggs. **Histograms of oriented gradients for human detection**. In Conference on Computer Vision and Pattern Recognition (CVPR), pages 886–893, 2005.

P. Dollár, Z. Tu, P. Perona, and S. Belongie. **Integral channel features**. In British Machine Vision Conference, pages 91.1–91.11, 2009.